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Question Paper Code : 41362

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2024

Third Semester

Computer Science and Engineering

MA 3354 – DISCRETE MATHEMATICS

(Common to : Computer Science and Engineering (Artificial Intelligence and Machine Learning)/Computer Science and Engineering (Cyber Security)/Computer and Communication Engineering/Artificial Intelligence and Data Science/Computer Science and Business Systems/Information Technology)

(Also common to PTMA 3354 for B.E. (Part-Time) — Third Semester — Computer Science and Engineering — Regulations 2023)

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write the converse, inverse and contra positive of the proposition "It snows whenever the wind blows from the northeast".

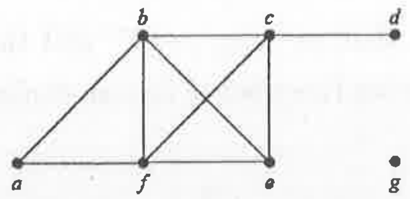
2. Comment on the proof strategy : In a conditional statement when the premise is false, irrespective of the truth value of the consequence, the truth value of the conditional statement is true.

3. Write the product rule in counting techniques.

4. Give a formula for the number of elements in the union of three sets. Name the principle that is useful in determining the number of elements in the union of sets.

5. Define graph. What is a loop in graph?

6. Determine the degree and neighborhood of the vertex b in the given graph.



7. Let $G = (Z_6, +)$. Determine whether $H = \{0, 2, 4\}$ is a subgroup of G?

8. Prove or disprove : The additive identity or zero element of a ring $(R, +, .)$ is unique.

9. Define partial ordering and partial ordered set.

10. Find the values of the Boolean function represented by $F(x, y) = x\bar{y}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences. (6)

(ii) Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset". (10)

Or

(b) (i) Express the statement "If a person is female and is a parent, then this person is someone's mother" as a logical expression involving predicates, quantifiers with a domain consisting of all people, and logical connectives. In addition express the same statement as a logical expression using the null quantification rule. (6)

(ii) Explain proof by contraposition, hence prove that if n is an integer and $3n + 2$ is odd, then n is odd. (10)

12. (a) (i) Why is mathematical induction a valid proof technique? Use mathematical induction to show that $1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$ for all nonnegative integers n. (8)

(ii) Find the solution to the recurrence relation

$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with the initial conditions $a_0 = 2$, $a_1 = 5$, and $a_2 = 15$. (8)

Or

(b) (i) How many permutations of the letters ABCDEFGH contain the string ABC? (4)

(ii) It is given that $a_n = 8a_{n-1} + 10^{n-1}$ and the initial condition $a_1 = 9$. Use generating functions to find an explicit formula for a_n. (12)

13. (a) Define graph isomorphism. What are graph invariant conditions for isomorphism? Construct the adjacency matrix for the given graphs. Determine whether the graphs G and H displayed in Figure 13(a) are isomorphic.

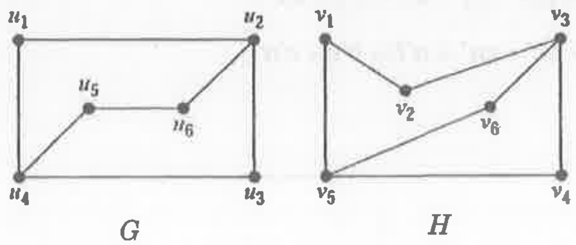


Figure 13(a)

Or

(b) Define the following : (6 × 2 + 4)

(i) connected graph

(ii) connected component of a graph

(iii) Euler circuit

(iv) Euler path

(v) Hamilton path

(vi) Hamilton circuit.

Determine whether the given graph Figure 13(b) has Euler path? If so, name the path, if not, give the reason.

Figure 13(b)

14. (a) (i) Let G be any group and $a \in G$. Define $\varphi_a : G \to G$ by $\varphi_a(x) = a * x * a^{-1}$ then prove that φ_a is a homomorphism. (8)

(ii) Let N be a normal subgroup of a group G. Prove that the mapping $\varphi : G \to G/N$ defined by $\varphi(a) = a * N$ is a homomorphism from G to G/N. (8)

Or

(b) Prove that, "If G is a finite group and H is a subgroup of G then $O(H)$ divides $O(G)$".

15. (a) State and prove Absorption law and isotonicity property on Lattice.

Or

(b) Let $(B, +, ., ')$ be a Boolean algebra. Show that the following

(i) $(a+b)(a'+c) = ac + a'b + bc$ (8)

(ii) $ab' + bc' + ca' = a'b + b'c + c'a$. (8)
